Guj-12-18

Mathematics
Time: 30 Minutes

(INTER PART II)-418-(I)
Code: 8191
OBJECTIVE

PAPER: II
Marks: 20

Note: You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two more circles will result in zero mark in that question. Attempt as many questions as given in objective type question paper and leave others blank.

1- 1- Range of $f(x) = x^2 + 1$ is
(A) R (B) $R - \{1\}$ (C) $R - \{-1\}$ (D) $[1, \infty)$

2- $\frac{e^x - e^{-x}}{2} =$
(A) $\sin x$ (B) $\cos x$ (C) $\sinh x$ (D) $\cosh x$

3- $\frac{d}{dx}\sqrt{x} =$
(A) $\sqrt{1}$ (B) 1 (C) $\frac{1}{2}\sqrt{x}$ (D) $\frac{1}{2\sqrt{x}}$

4- $\frac{d}{dx}\cos x^2 =$
(A) $\sin x^2$ (B) $\sin x^2$ (C) $-2x\sin x^2$ (D) $2x\sin x$

5- If $f(x)$ has maximum value at $x = c$, then $f'(c) = 0$ but $f''(c)$ is
(A) negative (B) positive (C) zero (D) undefined

6- $\frac{d}{dx}e^{f(x)} =$
(A) $e^{f(x)}$ (B) $f(x)e^{f(x)}$ (C) $e^{f(x)}f'(x)$ (D) $f(x)e^{f(x)-1}$

7- $\int 2^x dx =$
(A) $\frac{2^{x+1}}{x-1}$ (B) $x\,2^{x-1}$ (C) $2^x \ell n2$ (D) $\frac{2^x}{\ell n2}$

8- $\int e^{ax}(af(x)+f'(x))dx =$
(A) $e^{ax}.af(x)$ (B) $e^{ax}.f'(x)$ (C) $e^{ax}.f(x)$ (D) $e^{ax}.af'(x)$

9- $\int(\ell nx)\frac{1}{x}dx =$
(A) $(\ell nx)^2$ (B) $\frac{(\ell nx)^2}{2}$ (C) $\frac{1}{x^2}$ (D) $\frac{1}{x^2}$

10- $\int_0^1 \frac{1}{1+x^2}dx =$
(A) 1 (B) $\frac{\pi}{4}$ (C) 0 (D) $\frac{\pi}{2}$

(Turn over)

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11- Coordinates of mid-point of $A(-1, 4), B(6, 2)$

(A) $(-7, 2)$ (B) $(7, -2)$ (C) $\left(\frac{5}{2}, 3\right)$ (D) $\left(3, \frac{5}{2}\right)$

12- If m_1, m_2 are slopes of perpendicular lines, then $m_1 m_2 =$

(A) 0 (B) -1 (C) 1 (D) undefined

13- If a line meets x and y axes at $2, 3$ units, then its equation is

(A) $2x + 3y = 0$ (B) $3x + 2y = 0$ (C) $\frac{x}{2} + \frac{y}{3} = 0$ (D) $\frac{x}{2} + \frac{y}{3} = 1$

14- If $P(7, -2)$ lies on circle with centre $(-5, 3)$, then its radius is

(A) 13 (B) $\sqrt{13}$ (C) 17 (D) $\sqrt{17}$

15- To find optimal solution we evaluate the objective function at

(A) one point (B) origin (C) some points (D) corner points

16- Length of Latus Rectum of Parabola $x^2 = 5y$ is

(A) 5 (B) 20 (C) $\frac{5}{4}$ (D) 10

17- For hyperbola value of eccentricity e is

(A) 1 (B) less than 1 (C) greater than 1 (D) 0

18- If $a = b$ then equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ represents

(A) ellipse (B) parabola (C) hyperbola (D) circle

19- Direction cosines of z-axis are

(A) $[1, 0, 0]$ (B) $[1, 1, 1]$ (C) $[0, 1, 0]$ (D) $[0, 0, 1]$

20- If $\vec{u} = \vec{v}$, then $\vec{u}.(\vec{v} \times \vec{w}) =$

(A) 0 (B) 1 (C) -1 (D) cannot be calculated

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Mathematics (INTER PART II)-418 PAPER: II

Time: 2:30 hours Marks: 80

SUBJECTIVE

Note: Section I is compulsory. Attempt any three (3) questions from Section II.

SECTION I

2. Write short answers to any EIGHT questions: (2 x 8 = 16)

i- Define implicit function also write one example.

ii- For $f(x) = \frac{x^3 - x}{x^2+1}$, determine whether given function is even or odd

iii- Prove that: $\lim_{n \to +\infty} \left(1 + \frac{1}{n}\right)^n = e$

iv- Find by definition the derivative of $x(x-3)$ with respect to 'x'.

v- Find the derivative of $(x^2+5)(x^3+7)$ w.r.t. 'x'

vi- If $y = x^4 + 2x^2 + 2$ prove that $\frac{dy}{dx} = 4x\sqrt{y-1}$

vii- Differentiate $x^2 + \frac{1}{x^2}$ w.r.t., $x - \frac{1}{x}$

viii- Calculate $\frac{d}{dx}(\cos\sqrt{x} + \sqrt{\sin x})$

ix- If $f(x) = \ell n\,(e^x + e^{-x})$. Find $f'(x)$

x- Find $\frac{dy}{dx}$ if $y = (\ell n \tanh x)$

xi- Find y_2 if $x = a\cos\theta$, $y = a\sin\theta$.

xii- Divide 20 into two parts so that the sum of their squares will be maximum.

3. Write short answers to any EIGHT questions: (2 x 8 = 16)

i- Using differentials find $\frac{dy}{dx}$ and $\frac{dx}{dy}$. $x^4 + y^2 = xy^2$

ii- Evaluate: $\int \frac{1}{1+\cos x}\, dx$.

iii- Evaluate: $\int \frac{e^x}{e^x + 3}\, dx$.

iv- Evaluate: $\int \tan^{-1} x\, dx$.

v- Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos t\, dt$

vi- Evaluate: $\int_0^3 \frac{dx}{x^2+9}$

vii- Find the area between the x-axis and the curve $y = \cos\frac{1}{2}x$ from $x = -\pi$ to π .

viii- Define differential equation.

ix- Solve the differential equation $\frac{dy}{dx} = \frac{y}{x^2}$

x- Solve $(e^x + e^{-x})\frac{dy}{dx} = e^x - e^{-x}$

xi- Define corner point.

xii- Graph $3x - 2y \geq 6$ in xy – plane.

(Turn over)

4. Write short answers to any NINE questions: (2 x 9 = 18)

i- By means of slope, prove that the points $(-1, -3)$; $(1, 5)$; $(2, 9)$ are collinear.

ii- Find an equation of horizontal line through $(7, -9)$

iii- Find an equation of the line through $(5, -8)$ and perpendicular to the join of $A(-15, -8)$, B(10, 7)

iv- Find the distance of the pt. $(6, -1)$ from the line $6x - 4y + 9 = 0$

v- Find the lines represented by $6x^2 - 19xy + 15y^2 = 0$

vi- Find the focus and directrix of parabola $x^2 = 5y$

vii- Convert $x + 8 - y^2 + 2y = 0$ into the standard form and find its vertex.

viii- Find an equation of the ellipse with vertices $(0, \pm 5)$, and eccentricity $\frac{3}{5}$.

ix- Find the focus and covertices of an ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$

x- Decide whether the triples $45^\circ, 45^\circ, 60^\circ$ are the direction angles of a vector or not.

xi- Find the projection of vector $\underline{a}$ along vector $\underline{b}$ and projection of $\underline{b}$ along $\underline{a}$,

if $\underline{a} = 3\hat{i} + \hat{j} - \hat{k}$, $\underline{b} = -2\hat{i} - \hat{j} + \hat{k}$

xii- If $\underline{v}$ is a vector for which $\underline{v}\cdot\hat{i} = 0$, $\underline{v}\cdot\hat{j} = 0$, $\underline{v}\cdot\hat{k} = 0$. Find $\underline{v}$.

xiii- Verify that the vectors $\underline{a}$ and $\underline{b}\times\underline{a}$ are perpendicular to each other, if

$\underline{a} = 3\hat{i} - \hat{j} + 5\hat{k}$, $\underline{b} = 4\hat{i} + 3\hat{j} - 2\hat{k}$

SECTION II

5- (a) Find the values 'm' and 'n' so that given function 'f' is continuous at $x = 3$ 5

$$f(x) = \begin{cases} mx, & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$$

(b) Differentiate: $\frac{\sqrt{x^2+1}}{\sqrt{x^2-1}}$ w.r.t. x 5

6- (a) Evaluate: $\int x^3 e^{5x} dx$ 5

(b) Find the angles of the triangle whose vertices are $A(-5, 4)$, $B(-2, -1)$ and $C(7, -5)$ 5

7- (a) Evaluate: $\int_{\pi/6}^{\pi/4} \cos^2\theta \cot^2\theta \, d\theta$ 5

(b) Minimize $f(x, y) = 3x + y$ subject to the constraints 5

$3x + 5y \geq 15$, $x + 6y \geq 9$ $x \geq 0$, $y \geq 0$

8- (a) Find length of the chord cut off from the line $2x + 3y = 13$ by the circle $x^2 + y^2 = 26$ 5

(b) Find the angle between the following vectors: $\underline{u} = 2\underline{i} - \underline{j} + \underline{k}$, $\underline{v} = -\underline{i} + \underline{j}$ 5

9- (a) Find the centre, foci, eccentricity, vertices and equations of directrices of hyperbola 5

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

(b) Find the volume of the tetrahedron whose vertices are 5

A(2, 1, 8) , B(3, 2, 9), C(2, 1 4), D(3, 3, 0)