

MATHEMATICS
GROUP : FIRST

DGK-12-1-23

OBJECTIVE

TIME: 30 MINUTES

MARKS: 20

NOTE: You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

QUESTION NO. 1

- 1 $\int e^{-x} (\cos x - \sin x) dx = \dots\dots\dots$
(A) $-e^{-x} \sin x + c$ (B) $e^{-x} \sin x + c$ (C) $e^x \cos x$ (D) $-e^x \cos x + c$
- 2 The order of differential equation $x^2 \frac{d^2y}{dx^2} + \frac{dy}{dx} + 2x = 0$ is
(A) 1 (B) 2 (C) 3 (D) 4
- 3 Vertical line passes through (5,4) is
(A) $y = 4$ (B) $x = 5$ (C) $y = 5$ (D) $y = -4$
- 4 Slope of line perpendicular to $3x - 4y + 5 = 0$ is
(A) $-4/3$ (B) $-3/4$ (C) $3/4$ (D) $4/3$
- 5 Coordinate of mid-point of A (-1, 4) and B (6, 2) is $\dots\dots\dots$
(A) (-7, 2) (B) (7, -2) (C) $(5/2, 3)$ (D) $(5/2, -5/2)$
- 6 Graph of $4y \geq 5$ will be $\dots\dots\dots$ half plane
(A) lower (B) right (C) upper (D) left
- 7 Directrix of $y^2 = 8x$ is
(A) $x + 2 = 0$ (B) $x - 2 = 0$ (C) $y + 2 = 0$ (D) $y - 2 = 0$
- 8 Vertices of the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ are $\dots\dots\dots$
(A) (0, ± 4) (B) (± 4 , 0) (C) (± 5 , 0) (D) (0, ± 5)
- 9 The center of circle $x^2 + y^2 - 6x + 4y + 13 = 0$ is
(A) (3, -2) (B) (-3, 2) (C) (-3, -2) (D) (3, 2)
- 10 An angle in the semi-circle is of measure $\dots\dots\dots$
(A) 30° (B) 90° (C) 45° (D) 60°
- 11 $\begin{bmatrix} k & i & j \end{bmatrix} = \dots\dots\dots$
(A) 1 (B) -1 (C) 0 (D) 3
- 12 If $\underline{U} = \underline{i} + \alpha \underline{j} - \underline{k}$ and $\underline{V} = 2\underline{i} + \underline{j} + \underline{k}$ are perpendicular then $\alpha = \dots\dots\dots$
(A) 1 (B) 2 (C) -1 (D) 0
- 13 $f(x) = x \quad \forall x \in \mathbb{R}$ is called $\dots\dots\dots$
(A) Constant function (B) Identity function (C) Non-linear function (D) Trigonometric function
- 14 $\lim_{x \rightarrow 0} (1-x)^{1/x} = \dots\dots\dots$
(A) e^x (B) ∞ (C) $e^{1/x}$ (D) e^{-1}
- 15 $\frac{d}{dx} (\tan x) = \dots\dots\dots$
(A) $\ln \cos x$ (B) $-\ln \cos x$ (C) $\sec^2 x$ (D) $-\sec^2 x$
- 16 If $f(x) = \sin x$ then $f'(\frac{\pi}{2}) = \dots\dots\dots$
(A) 0 (B) 1 (C) 2 (D) -1
- 17 $\frac{d}{dx} (\cosh 2x) = \dots\dots\dots$
(A) $\cosh 2x$ (B) $2 \cosh 2x$ (C) $2 \sinh 2x$ (D) $\sinh 2x$
- 18 For a stationary point of function we have $f'(x) = \dots\dots\dots$
(A) 0 (B) Positive (C) Negative (D) ∞
- 19 If $v = x^3$ then differential of v is
(A) $3x^2$ (B) $3x^2 dv$ (C) $x^3 dx$ (D) $3x^2 dx$
- 20 $\int \frac{\sec^2 x}{\tan x} dx = \dots\dots\dots$
(A) $\tan x + c$ (B) $-\cot x + c$ (C) $\ln(\tan x) + c$ (D) $\sec x + c$

D

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SECTION-I

QUESTION NO. 2 Write short answers any Eight (8) of the following

16

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| i | Express the area A of a circle as a function of its circumference C. |
| ii | For any real valued function of $f(x) = 2x + 1$, find $f \circ f(x)$. |
| iii | Evaluate $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin \theta}$ |
| iv | Differentiate $(x - 5)(3 - x)$ w.r.t x |
| v | Find $\frac{dy}{dx}$ if $xy + y^2 = 2$ |
| vi | Find $\frac{dy}{dx}$ if $y = x \cos y$ |
| vii | Find $f'(x)$ if $f(x) = e^x(1 + \ln x)$ |
| viii | Find y_2 if $x^2 + y^2 = a^2$ |
| ix | Apply Maclaurin series expansion to prove that $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$ |
| x | Find the extreme values for the function $f(x) = 5x^2 - 6x + 2$ |
| xi | Define convex region. |
| xii | Graph the solution set of the inequality $5x - 4y \leq 20$ |

QUESTION NO. 3 Write short answers any Eight (8) of the following

16

| | |
|------|---|
| i | Evaluate $\int \frac{dx}{\sqrt{x+1}-\sqrt{x}}$ |
| ii | Evaluate $\int \frac{adt}{2\sqrt{at+b}}$ |
| iii | Find $\int x \ln x \, dx$ |
| iv | Evaluate the definite integral $\int_{-6}^2 \sqrt{3-x} \, dx$ |
| v | Evaluate $\int \frac{2x}{x^2-a^2} \, dx$, $x > a$ |
| vi | Evaluate $\int (x+1)(x-3) \, dx$ |
| vii | Evaluate $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) \, dx$, $x > 0$ |
| viii | Define equal Vectors. |
| ix | Find the unit vector in the direction of the vector $\underline{v} = 2\underline{i} + 6\underline{j}$ |
| x | Let $\underline{A} = (2, 5)$, $\underline{B} = (-1, 1)$ Find \underline{AB} |
| xi | Write two properties of Dot Product. |
| xii | Define cross product of two vectors and give its geometrical meanings. |

QUESTION NO. 4 Write short answers any Nine (9) of the following

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| i | The points A (-5, -2) and B(5, -4) are ends of diameter of Circle, Find the Center and radius of Circle. |
| ii | The coordinates of P are (-6, 9), the axes are translated through point $O'(-3, 2)$, Find coordinate of P referred to new axes. |
| iii | By means of slopes, show that (4, -5), (7, 5) and (10, 15) lie on same line. |
| iv | Find equation of line whose x-intercept is -3, y-intercept is 4. |
| v | Convert $15y - 8x + 3 = 0$ into normal and slope intercept form. |
| vi | Check whether the lines $4x - 3y - 8 = 0$, $3x - 4y - 6 = 0$ and $x - y - 2 = 0$ are concurrent. |
| vii | Find lines represented by $6x^2 - 19xy + 15y^2 = 0$ |
| viii | Find centre and radius of circle $5x^2 + 5y^2 + 24x + 36y + 10 = 0$ |
| ix | Find equation of circle with centre $(\sqrt{2}, -3\sqrt{3})$ and radius $2\sqrt{2}$ |
| x | Write equation of tangent to $3x^2 + 3y^2 + 5x - 13y + 2 = 0$ at $\left(1, \frac{10}{3}\right)$ |
| xi | Find focus and vertex of parabola $y^2 = -8(x - 3)$ |
| xii | Find equation of ellipse having centre (0, 0), focus at (0, -3) and one vertex at (0, 4) |
| xiii | Find eccentricity and vertices of hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ |

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SECTION-II

Note: Attempt any Three questions from this section

10 x 3 = 30

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| Q.5-(A) | Find the values m and n , so that the given function is continuous at $x = 3$ $f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$ |
| (B) | If $y = \sqrt{x} - \frac{1}{\sqrt{x}}$ show that $2x \frac{dy}{dx} + y = 2\sqrt{x}$ |
| Q.6-(A) | Evaluate the indefinite integral $\int \sqrt{x^2 - a^2} dx$ |
| (B) | Find the equation of the medians of triangle whose vertices are $A(-3,2)$, $B(5,4)$ and $C(3,-8)$ |
| Q.7-(A) | Evaluate $\int_0^{\pi/4} (1 + \cos^2 \theta) \tan^2 \theta d\theta$ |
| (B) | Maximize $f(x, y) = x + 3y$; subject to the constraints $2x + 5y \leq 30$ $5x + 4y \leq 20$ $x \geq 0, y \geq 0$ |
| Q.8-(A) | Find $f'(x)$ if $f(x) = \sqrt{\ln(e^{2x} + e^{-2x})}$ |
| (B) | Write an equation of the circle that passes through the points $A(4,5)$, $B(-4,-3)$, $C(8,-3)$ |
| Q.9-(A) | Find the focus, vertex and directrix of the parabola $x + 8 - y^2 + 2y = 0$ |
| (B) | Prove that angle in a semi circle is a right angle. |