

Roll No. 754990 to be filled in by the candidate.

(For all sessions)

Paper Code

8

1

9

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Mathematics (Objective Type)

RWP-21

Time: 30 Minutes

Marks:20

NOTE: Write answers to the questions on objective answer sheet provided. Four possible answers A,B,C & D to each question are given. Which answer you consider correct, fill the corresponding circle A,B,C or D given in front of each question with Marker or pen ink on the answer sheet provided.

1-1. If $g(x) = \frac{1}{x^2}$, $x \neq 0$ then $g \circ g(x)$ equals.

(A) x (B) x^2 (C) x^4 (D) x^3

2. $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta}$ equals.

(A) zero

(B) 1

(C) 2

(D) 3

3. The derivative of $\sqrt{x}$ at $x = 1$ is:

(A) $\frac{1}{2}$

(B) 2

(C) 1

(D) $-\frac{1}{2}$

4. $\frac{d}{dx} \left[\frac{1}{g(x)} \right]$ equals.

(A) $\frac{1}{g^2(x)}$ (B) $\frac{-g'(x)}{(g(x))^2}$ (C) $-g(x)$ (D) $\frac{1}{g(x)}$

5. If $y = 5e^x$ then y_3 equals.

(A) $25e^x$ (B) $75e^x$ (C) $15e^x$ (D) $5e^x$

6. If $f(x+h) = \cos(x+h)$ then $f'(x)$ equals.

(A) $\cos x$ (B) $-\cos x$ (C) $-\sin x$ (D) $\sin x$

7. Inverse of $\int \dots dx$ is:

(A) $\frac{d}{dy}$ (B) $\frac{d}{dx}$ (C) $\frac{dy}{dx}$ (D) $\frac{dx}{dy}$

8. $\int_a^b f(x) dx$ equals:

(A) $-\int_b^a f(x) dx$ (B) $\int_{-b}^a f(x) dx$ (C) $\int_b^{-a} f(x) dx$ (D) $\int_a^{-b} f(x) dx$

9. The general solution of $\frac{dy}{dx} = \frac{-y}{x}$ is:

(A) $xy = c$ (B) $x^2 y^2 = c$ (C) $\frac{x}{y} = c$ (D) $\frac{y}{x} = c$

10. $\int e^{-x} (\cos x - \sin x) dx$ equals:

- (A) $-\bar{e}^x \sin x + c$ (B) $\bar{e}^x \cos x + c$ (C) $\bar{e}^x + c$ (D) $\bar{e}^x \sin x + c$

11. The distance of point (3,7) from x-axis is:

- (A) 3 (B) 7 (C) -3 (D) -7

12. Slope of Y-axis is:

- (A) zero (B) 1 (C) 2 (D) undefined

13. Equation of horizontal line through (7,-9) is:

- (A) $y = -9$ (B) $y = 7$ (C) $x = -9$ (D) $x = 7$

14. (0,2) is solution of inequality.

- (A) $3x + 5y > 7$ (B) $3x + 5y < 7$ (C) $x < 0$ (D) $x > 0$

15. Centre of circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is:

- (A) (g, f) (B) $(-g, f)$ (C) $(0, 0)$ (D) $(-g, -f)$

16. Equation of Latus rectum of parabola $x^2 = 4ay$ is:

- (A) $y = -a$ (B) $y = a$ (C) $x = -a$ (D) $x = a$

17. Vertices of $\frac{x^2}{16} - \frac{y^2}{25} = 1$ are:

- (A) $(0, \pm 4)$ (B) $(0, \pm 5)$ (C) $(\pm 4, 0)$ (D) $(\pm 5, 0)$

18. The non zero vectors $\underline{a}$ and $\underline{b}$ are parallel if $\underline{a} \times \underline{b}$ is:

- (A) zero (B) 1 (C) 2 (D) 3

19. $\cos \theta$ equals:

- (A) $\underline{a} \cdot \underline{b}$ (B) $\underline{a} \times \underline{b}$ (C) $|\underline{a} \times \underline{b}|$ (D) $\underline{\hat{a}} \cdot \underline{\hat{b}}$

20. If any two vectors of scalar triple product are equal then its value is:

- (A) -1 (B) zero (C) 1 (D) 2

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Mathematics (Essay Type)

Time: 2:30 Hours

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Section -I

Marks: 80

2. Write short answers of any eight parts from the following.

2x8=16

- If $f(x) = x^2 - x$, find (a). $f(-2)$ (b). $f(x-1)$
- Find $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 + x - 6}$.
- Find $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin \theta}$.
- Differentiate w.r.t "x" $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$.
- Find $\frac{dy}{dx}$ if $3x + 4y + 7 = 0$.
- Differentiate w.r.t "x" $\cot^{-1}\left(\frac{x}{a}\right)$.
- If $y = x^2 \cdot e^{-x}$, then find $\frac{d^2 y}{dx^2}$.
- If $y = \log_{10}(ax^2 + bx + c)$, then find $\frac{dy}{dx}$.
- Apply Maclaurin series, Prove that $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$
- If $f(x) = \sqrt{x+1}$ and $g(x) = \frac{1}{x^2}$, then find (a). $(f \circ g)(x)$ (b). $(g \circ f)(x)$.
- Find the intervals in which $f(x)$ is increasing or decreasing $f(x) = \cos x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

3. Write short answers of any eight parts from the following.

2x8=16

- Using differential find $\frac{dy}{dx}$, if $x^2 + 2y^2 = 16$.
- Evaluate $\int x\sqrt{x^2-1} dx$.
- Evaluate $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx$.
- Evaluate $\int \sin^2 x dx$.
- Evaluate $\int \frac{ax+b}{ax^2+2bx+c} dx$.
- Evaluate $\int e^{3x} \left(\frac{3 \sin x - \cos x}{\sin^2 x}\right) dx$.
- Solve $\frac{dy}{dx} = \frac{y^2+1}{e^{-x}}$.
- Find an equation of the vertical line through (-5,3).
- Find an equation of the line through (-5,-3), (9,-1)
- Convert $4x + 7y - 2 = 0$ in normal form.
- Find the area below the curve $y = 3\sqrt{x}$ and above the x-axis between $x = 1$ and $x = 4$.
- Find the mid point of the line segment joining the points A(3,1), B(-2,-4).

4. Write short answers of any nine parts from the following.

2x9=18

- Graph the solution set by shading of inequality $5x - 4y \leq 20$.
- Find equation of circle with centre at $(\sqrt{2}, -3\sqrt{3})$ and radius $2\sqrt{2}$.
- Write equation of tangent to the circle $3x^2 + 3y^2 + 5x - 13y + 2 = 0$ at $(1, \frac{10}{3})$.

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- iv. Find vertex of $x^2 - 4x - 8y + 4 = 0$.
- v. Find point of intersection of conics $3x^2 - 4y^2 = 12$ and $3y^2 - 2x^2 = 7$.
- vi. Find equation of parabola whose focus is $F(-3,4)$ and directrix is $3x - 4y + 5 = 0$.
- vii. Find the unit vector in the same direction of vector $\underline{V} = [3, -4]$.
- viii. If $\overline{AB} = \overline{CD}$ find the co-ordinate of the point A when points B, C, D are (1,2), (-2,5) and (4,11) respectively
- ix. Find $|3\underline{v} + \underline{w}|$ if $\underline{u} = \underline{i} + 2\underline{j} - \underline{k}$, $\underline{v} = 3\underline{i} - 2\underline{j} + 2\underline{k}$, $\underline{w} = 5\underline{i} - \underline{j} + 3\underline{k}$.
- x. Find a vector of length 5 in the direction opposite that of $\underline{v} = \underline{i} - 2\underline{j} + 3\underline{k}$.
- xi. Compute $\underline{b} \times \underline{a}$ if $\underline{b} = \underline{i} - \underline{j} + \underline{k}$, $\underline{a} = 2\underline{i} + \underline{j} - \underline{k}$.
- xii. Find the work done if the point at which the constant force $\overline{F} = 4\underline{i} + 3\underline{j} + 5\underline{k}$ is applied to an object, moves from $p_1(3,1,-2)$ to $p_2(2,4,6)$.
- xiii. If $\underline{a} + \underline{b} + \underline{c} = 0$ then prove that $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$.

Section -II

Note: Attempt any three questions from the following.

10x3=30

5. (a) If $f(x) = \begin{cases} 3x-1 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ 2x & \text{if } x > 1 \end{cases}$, then show $f(x)$ is continuous at $x = 1$.

(b) If $x = \frac{a(1-t^2)}{1+t^2}$, $y = \frac{2bt}{1+t^2}$, then find $\frac{dy}{dx}$.

6. (a) Find the approximate increase in the volume of a cube of the length of its each edge changes from 5 to 5.02.

- (b) Determine the value of P such that the lines $2x - 3y - 1 = 0$, $3x - y - 5 = 0$ and $3x + py + 8 = 0$ meet at a point.

7. (a) Evaluate $\int_2^3 \left(x - \frac{1}{x} \right)^2 dx$.

- (b) Minimize $z = 2x + y$ subject to the constraints $x + y \geq 3$, $7x + 5y \leq 35$, $x \geq 0$, $y \geq 0$.

8. (a) Write equations of two tangents from (2,3) to the circle $x^2 + y^2 = 9$.

- (b) Prove by vector method $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$.

9. (a) Show that $\cos(x+h) = \cos x - h \sin x - \frac{h^2}{2!} \cos x + \frac{h^3}{3!} \sin x + \dots$

- (b) Show that an equation of the parabola with focus at $(a \cos \alpha, a \sin \alpha)$ and

directrix $x \cos \alpha + y \sin \alpha + a = 0$ is $(x \sin \alpha - y \cos \alpha)^2 = 4a(x \cos \alpha + y \sin \alpha)$.

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