MATHEMATICS PAPER-II MTN-G1-21 TIME ALLOWED: 30 Minutes

GROUP-I MTN-G1-21 **OBJECTIVE** MAXIMUM MARKS: 20

Note: **You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that bubble in front of that question number, on bubble sheet. Use marker or pen to fill the bubbles. Cutting or filling two or more bubbles will result in zero mark in that question. No credit will be awarded in case BUBBLES are not filled. Do not solve question on this sheet of OBJECTIVE PAPER.**

MTN-GI-21

Q.No.1

(1) When the expression $\sqrt{a^2-x^2}$ involves in integration substitute, is:
(A) $x = a\sin\theta$ (B) $a\sec\theta$ (C) $a\tan\theta$ (D) $a=\sin\theta$

(2) $\int_0^3 \frac{1}{\sqrt{9-x^2}}dx =$ ______ (A) $\frac{2}{\pi}$ (B) $\frac{-2}{\pi}$ (C) $\frac{-\pi}{2}$ (D) $\frac{\pi}{2}$

(3) Which of the following is not a solution of the system of inequalities
$x+2y\le 8,\ 2x-3y\le 6,\ 2x+y\ge 2 \quad x\ge 0,\ y\ge 0$
(A) (1, 0) (B) (8, 0) (C) (0, 4) (D) (3, 0)

(4) When axes are translated, the coordinates of the point $(-6, 9)$ are changed into $(-3, 7)$, find the point through which axes are translated:
(A) $(-3, 2)$ (B) $(3, -2)$ (C) $(7, -3)$ (D) $(-9, 6)$

(5) The equation of horizontal line passing through $(-5, 3)$ is:
(A) $x+5=0$ (B) $-5x+3y=0$ (C) $3x-5y=0$ (D) $y-3=0$

(6) A line passes through $(1, 5)$ and $(k, 7)$ has a slope k, the values of k is:
(A) -1 and 2 (B) 3 and -2 (C) $2, -3$ (D) $-1, -2$

(7) The focus of the parabola $y^2 = 4ax$ is:
(A) $(a, 0)$ (B) $(0, a)$ (C) $(-a, 0)$ (D) $(0, -a)$

(8) The eccentricity of $\frac{y^2}{4} - x^2 = 1$ equals: (A) $\frac{-2}{\sqrt{5}}$ (B) $\frac{2}{\sqrt{5}}$ (C) $\frac{-\sqrt{5}}{2}$ (D) $\frac{\sqrt{5}}{2}$

(9) Conic are the curves obtained by cutting a right circular cone by:
(A) Sphere (B) A line (C) A plane (D) A curve

(10) If $\underline{a}$ and $\underline{b}$ are two non-zero vectors, then $\underline{a}\times\underline{b} =$ ______
(A) $-\underline{b}\times\underline{a}$ (B) $\underline{a}\cdot\underline{b}$ (C) $-\underline{a}\times-\underline{b}$ (D) $\underline{b}\times\underline{a}$

(11) Angle between the vectors $\underline{i}+\underline{j},\ \underline{i}-\underline{j}$ is: (A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) 0

(12) Projection of $\underline{a}$ along $\underline{b}$ is:
(A) $\hat{a}\cdot\hat{b}$ (B) $\underline{a}-\underline{b}$ (C) $\underline{a}\cdot\hat{b}$ (D) $\hat{a}\cdot\underline{b}$

(13) If $f(x)=\sqrt{x+4}$ then $f(x^2+4)$ is equal to:
(A) $\sqrt{x^2+8}$ (B) $\sqrt{x^2-8}$ (C) $\sqrt{x-8}$ (D) x^2-8

(14) The function $f(x)=\frac{2+3x}{2x}$ is not continuous at: (A) $x=-3$ (B) $x=-2/3$ (C) $x=1$ (D) $x=0$

(15) $\lim_{x\to a}\frac{f(x)-f(a)}{x-a} =$ ______ (A) $f'(x)$ (B) $f'(a)$ (C) $f'(0)$ (D) $f'(x-a)$

(16) $f(x)=x^{2/3}$, then $f'(8)=$ ______ (A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) 3

(17) The derivative of $\frac{x^3+2x^2}{x^3}$ equals: (A) $\frac{2}{x^2}$ (B) $\frac{-2}{x^2}$ (C) $\frac{1}{2x^2}$ (D) $\frac{-1}{2x^2}$

(18) If $f(x)=\tan^{-1}x$, then $f'(\cot x)$ is equal to:
(A) $\frac{1}{1+x^2}$ (B) $\sin^2 x$ (C) $\cos^2 x$ (D) $sex^2 x$

(19) $f(x+\delta x)=$ ______
(A) $f'(x)dx$ (B) $f(x)-f'(x)dx$ (C) $f(x)+f'(x)dx$ (D) $f(x)dx$

(20) $\int\frac{a}{x\sqrt{x^2-1}}dx =$ ______

(A) $a\tan^{-1}x$ (B) $-a\,\mathrm{cosec}^{-1}x+c$ (C) $-a\sec^{-1}x+c$ (D) $\frac{1}{a}\sec^{-1}x+c$

MATHEMATICS PAPER-II
GROUP-I

TIME ALLOWED: 2.30 Hours
MAXIMUM MARKS: 80

SUBJECTIVE

NOTE: Write same question number and its part number on answer book, as given in the question paper.

MTN-G1-21

SECTION-I

2. **Attempt any eight parts.** **$8 \times 2 = 16$**

(i) Determine whether the function $f(x) = \sin x + \cos x$ is even or odd.

(ii) With out finding the inverse, state domain and range of f^{-1} where $f(x) = \dfrac{x-1}{x-4}$ $x \neq 4$

(iii) Evaluate the limit $\lim\limits_{x \to 2} \dfrac{x^3 - 8}{x^2 + x - 6}$ by using algebraic techniques.

(iv) Express the Limit $\lim\limits_{x \to 0} (1 + 2x^2)^{\frac{1}{x^2}}$ in terms of e.

(v) Find the derivative of $(x+4)^{\frac{1}{3}}$ by definition.

(vi) Differentiate $x^2 - \dfrac{1}{x^2}$ w.r.t x^4.

(vii) If $y = \ell n\,(x + \sqrt{x^2 + 1})$ then find $\dfrac{dy}{dx}$

(viii) If $y = x^2 \cdot e^{-x}$ then find y_2

(ix) If $x = 1 - t^2$ and $y = 3t^2 - 2t^3$ then find $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$

(x) If $f(x) = 4 - x^2$, $x \in (-2, 2)$ then find interval in which $f(x)$ is increasing or decreasing.

(xi) Prove that $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$

(xii) If $y = \sin 3x$ then find y_4

3. **Attempt any eight parts.** **$8 \times 2 = 16$**

(i) Using differentials find $\dfrac{dy}{dx}$ and $\dfrac{dx}{dy}$ if $x^2 + 2y^2 = 16$

(ii) Evaluate $\int \cos 3x \sin 2x \, dx$

(iii) Evaluate $\int \dfrac{x^2}{4 + x^2} \, dx$

(iv) Evaluate $\int x \, \ell nx \, dx$

(v) Evaluate $\int \dfrac{(a-b)x}{(x-a)(x-b)} \, dx$, $a > b$

(vi) Evaluate $\int_0^3 \dfrac{dx}{x^2 + 9}$

(vii) Solve the differential equation $y\,dx + x\,dy = 0$

(viii) Evaluate $\int \sec x \, dx$

(ix) Find K so that the line joining $A(7, 3)$, $B(K, -6)$ and the line joining $C(-4, 5)$, $D(-6, 4)$ are parallel.

(x) Find whether the given point $P(5, 8)$ lies above or below the line $2x - 3y + 6 = 0$

(xi) Determine value of P such that the lines $2x - 3y - 1 = 0$, $3x - y - 5 = 0$ and $3x + Py + 8 = 0$ meet at a point.

(xii) Find the lines represented by $3x^2 + 7xy + 2y^2 = 0$

4. Attempt any nine parts. $9 \times 2 = 18$

(i) Graph the solution set of linear inequality in xy-plane $3x - 2y \geq 6$

(ii) Find the equation of a circle with ends of a diameter at $(-3, 2)$ and $(5, -6)$

(iii) Find the centre and radius of a circle $4x^2 + 4y^2 - 8x + 12y - 25 = 0$

(iv) Write down the equation of normal to the circle $x^2 + y^2 = 25$ at $(4, 3)$

(v) Find the vertex and directrix of $x^2 = 4(y - 1)$

(vi) Write the equation of parabola with focus $(-3, 1)$ and directrix $x - 2y - 3 = 0$

(vii) Find the equation of hyperbola with Foci $(\pm 5, 0)$ and vertex is $(3, 0)$

(viii) Find the magnitude of vector $\underline{u} = \underline{i} + \underline{j}$

(ix) Find a unit vector in the direction of $\underline{v} = \underline{i} + 2\underline{j} - \underline{k}$

(x) Find the direction cosines of $\underline{v} = 3\underline{i} - \underline{j} + 2\underline{k}$

(xi) If $\underline{u} = [2, -3, 1]$, $\underline{v} = [2, 4, 1]$ find the cosine of angle θ between $\underline{u}$ and $\underline{v}$

(xii) If $\underline{a} \times \underline{b} = 0$ and $\underline{a} \cdot \underline{b} = 0$, what conclusion can be drawn about $\underline{a}$ or $\underline{b}$?

(xiii) Find α so that $\underline{i} - \underline{j} + \underline{k}$, $\underline{i} - 2\underline{j} - 3\underline{k}$ and $3\underline{i} - \alpha\underline{j} + 5\underline{k}$ are coplanar.

SECTION-II

NOTE: Attempt any three questions. $3 \times 10 = 30$

5.(a) If $f(x) = \begin{cases} \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & x \neq 2 \\ K, & x = 2 \end{cases}$ Find the value of K so that f is continuous at $x = 2$

(b) If $x = a\cos^3\theta$, $y = b\sin^3\theta$, show that $a\dfrac{dy}{dx} + b\tan\theta = 0$

6.(a) Evaluate the integral $\displaystyle\int \frac{\sqrt{2}}{\sin x + \cos x}\, dx$

(b) Find the condition that the lines $y = m_1x + c_1$, $y = m_2x + c_2$, $y = m_3x + c_3$ are concurrent.

7. (a) Evaluate $\displaystyle\int_0^{\pi/4} \frac{\sin x - 1}{\cos^2 x}\, dx$

(b) Maximize $f(x, y) = 2x + 5y$ subject to constraints $2y - x \leq 8$, $x - y \leq 4$, $x \geq 0$, $y \geq 0$

8. (a) Write equations of two tangents from $(2, 3)$ to the circle $x^2 + y^2 = 9$

(b) By using vectors prove that $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

9.(a) If $y = (\cos^{-1} x)^2$, prove that $(1 - x^2)y_2 - xy_1 - 2 = 0$

(b) Show that an equation of parabola with focus at $(a\cos\alpha, a\sin\alpha)$ and directrix $x\cos\alpha + y\sin\alpha + a = 0$ is $(x\sin\alpha - y\cos\alpha)^2 = 4a(x\cos\alpha + y\sin\alpha)$