

MATHEMATICS

GROUP : FIRST

NOTE: You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

QUESTION NO. 1

- 1 $\frac{d}{dx} (\cos^{-1} \frac{x}{a}) = \dots\dots\dots$
(A) $\frac{1}{1-x^2}$ (B) $\frac{1}{1+x^2}$ (C) $\frac{-1}{\sqrt{a^2-x^2}}$ (D) $\frac{1}{\sqrt{a^2-x^2}}$
- 2 If $y = \ln(\sin x)$, then $\frac{dy}{dx}$ is
(A) $\tan x$ (B) $\cot x$ (C) $-\tan x$ (D) $-\cot x$
- 3 The minimum value of the function $f(x) = x^2 + 2x - 3$ is at $x = \dots\dots\dots$
(A) -3 (B) 1 (C) 0 (D) -1
- 4 $\int x^{-1} dx = \dots\dots\dots$
(A) $0 + c$ (B) $-x^{-2} + c$ (C) $\frac{x^{-2}}{-2} + c$ (D) $\ln x + c$
- 5 $\int \frac{1}{1+\cos x} dx = \dots\dots\dots$
(A) $\frac{1}{2} \tan \frac{x}{2}$ (B) $\tan \frac{x}{2}$ (C) $\cot \frac{x}{2}$ (D) $\frac{1}{2} \cot \left(\frac{x}{2}\right)$
- 6 $\int_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} \frac{dx}{\sqrt{1-x^2}} = \dots\dots\dots$
(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{12}$
- 7 The order of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 3x = 0$ is
(A) 1 (B) 2 (C) 0 (D) 3
- 8 The solution set of inequality $2x - 3 \geq 0$ is
(A) $\left[\frac{3}{2}, \infty\right]$ (B) $\left[\frac{2}{3}, \infty\right]$ (C) $\left[\frac{2}{3}, \infty\right]$ (D) $\left[\frac{3}{2}, 0\right]$
- 9 Perpendicular distance of the point $P(6, -1)$ from the line $3x + 4y + 1 = 0$ is
(A) 3 (B) 11 (C) 2 (D) 4
- 10 The coordinates of the point that divides the join of $A(-6, 3)$ and $B(5, -3)$ in the ratio $2 : 3$ externally
(A) $\left(-\frac{8}{3}, 1\right)$ (B) $\left(\frac{8}{5}, -1\right)$ (C) $(-28, 13)$ (D) $(28, -13)$
- 11 If coordinates of the mid points of the sides of a triangle are $(3, 2)$, $(2, 3)$ and $(1, -1)$, then the area of the triangle is
(A) 10 sq. units (B) 6 sq. units (C) 11 sq. units (D) 5 sq. units
- 12 The latus rectum of a parabola $y^2 = 4ax$ is
(A) $y = -a$ (B) $x = -a$ (C) $y = a$ (D) $x = a$
- 13 Condition that line $y = mx + c$ is tangent to the circle $x^2 + y^2 = a^2$ is
(A) $c = \pm m\sqrt{1+a^2}$ (B) $c = \pm m\sqrt{1-a^2}$ (C) $c = \pm a\sqrt{1-m^2}$ (D) $c = \pm a\sqrt{1+m^2}$
- 14 The projection of $\underline{u} = a\underline{i} + b\underline{j} + c\underline{k}$ along \underline{i} is
(A) 0 (B) b (C) a (D) c
- 15 A constant force \underline{F} acting on a body, displaces it from A to B. The work done by \underline{F} is
(A) $\underline{F} \cdot \underline{AB}$ (B) $\underline{F} \times \underline{AB}$ (C) $-\underline{F} \times \underline{AB}$ (D) $-\underline{F} \cdot \underline{AB}$
- 16 The angle between the vectors $4\underline{i} + 2\underline{j} - \underline{k}$ and $-\underline{i} + \underline{j} - 2\underline{k}$ is
(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π
- 17 The coordinates of vertices of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is
(A) $(\pm a, 0)$ (B) $(0, \pm b)$ (C) $(0, \pm a)$ (D) $(\pm b, 0)$
- 18 If $f(x) = -2x+6$, then $f^{-1}(x) = \dots\dots\dots$
(A) $6-2x$ (B) $\frac{6-x}{2}$ (C) $\frac{2}{6-x}$ (D) $2x-6$
- 19 $\lim_{x \rightarrow 0} (1+3x)^{2/x} = \dots\dots\dots$
(A) e^2 (B) e^8 (C) e^6 (D) e^4
- 20 If $f(x) = \tan x$, then $f\left(\frac{\pi}{4}\right) = \dots\dots\dots$
(A) 1 (B) $\frac{1}{2}$ (C) 2 (D) $\frac{1}{3}$

QUESTION NO. 2 Write short answers any Eight (8) of the following

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1	Find the Domain and Range of $f(x) = x$
2	Determine whether the function $f(x) = \frac{3x}{x^2+1}$ is even or odd
3	For the functions $f(x) = 3x^4 - 2x^2$, $g(x) = \frac{2}{\sqrt{x}}$ find $f \circ g(x)$ and $g \circ f(x)$
4	Evaluate $\lim_{x \rightarrow \infty} \frac{5x^4 - 10x^2 + 1}{3x^3 + 10x^2 + 50}$
5	Find by definition the derivative of $\frac{1}{x^3}$
6	Differentiate $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$ w.r.t x
7	Find $\frac{dy}{dx}$ if $x^2 - 4xy - 5y = 0$
8	Differentiate $\sin x$ w.r.t $\cot x$
9	For $f(x) = \ln \sqrt{e^{2x} + e^{-2x}}$; find $f'(x)$
10	Find y_1 if $x^3 - y^3 = a^3$
11	Find extreme values of $f(x) = 2x^3 - 2x^2 - 36x + 3$
12	Find $\frac{dy}{dx}$ if $y = \ln(\tan h x)$

QUESTION NO. 3 Write short answers any Eight (8) of the following

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1	Find dy if $y = x^2 + 2x$, when x changes from 2 to 1.8
2	Evaluate $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx$ ($x > 0$)
3	Evaluate $\int \frac{\cot \sqrt{x}}{\sqrt{x}} dx$
4	Evaluate $\int e^x (\cos x + \sin x) dx$
5	Evaluate $\int_1^2 \frac{x}{x^2+2} dx$
6	Evaluate $\int_0^{\pi/3} \cos^2 x \cdot \sin x dx$
7	Find the area between the x-axis and the curve $y = x^2 + 1$ from $x = 1$ to $x = 2$
8	Solve the differential equation $\frac{dy}{dx} = -y$
9	Show that the points $A(0,2)$, $B(\sqrt{3}, -1)$ and $C(0,-2)$ are vertices of a right triangle
10	Find an equation of the line through $(-4, -6)$ and perpendicular to a line having slope $-3/2$
11	Find whether the point $(5,8)$ lies above or below the line $2x - 3y + 6 = 0$
12	Find the lines represented by $20x^2 + 17xy - 24y^2 = 0$

QUESTION NO. 4 Write short answers any Nine (9) of the following

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1	Graph the solution set of $2x + y \leq 6$
2	Find equation of circle with ends of a diameter at $(-3, 2)$ and $(5, -6)$
3	Find centre and radius of circle $5x^2 + 5y^2 + 14x + 12y - 10 = 0$
4	Find vertex and directrix of parabola $x^2 = -16y$
5	Find an equation of parabola whose focus is $F(-3,4)$ and directrix $3x - 4y + 5 = 0$
6	Find foci and vertices of Hyperbola $\frac{y^2}{16} - \frac{x^2}{49} = 1$
7	Find centre and eccentricity of $\frac{x^2}{4} - \frac{y^2}{9} = 1$
8	Find magnitude of vector $\underline{u} = \underline{i} + \underline{j}$
9	Find a unit vector in the direction of $\underline{v} = [-2, 4]$
10	Find a vector of length 5 in the direction opposite that of $\underline{v} = \underline{i} - 2\underline{j} + 3\underline{k}$
11	If \underline{v} is a vector for which $\underline{v} \cdot \underline{i} = 0$, $\underline{v} \cdot \underline{j} = 0$, $\underline{v} \cdot \underline{k} = 0$ Find \underline{v}
12	Compute $\underline{a} \times \underline{b}$ if $\underline{a} = -4\underline{i} + \underline{j} - 2\underline{k}$, $\underline{b} = 2\underline{i} + \underline{j} + \underline{k}$
13	Find the value of $3\underline{j} \cdot \underline{k} \times \underline{i}$

Note: Attempt any Three questions from this section

10 x 3 = 30

Q.5- (A)	If $f(x) = \begin{cases} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases}$ Find k so that f(x) is continuous at x = 2
(B)	Prove that $y \frac{dy}{dx} + x = 0$ if $x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$
Q.6- (A)	Evaluate $\int \frac{x \sin^{-1}x}{\sqrt{1-x^2}} dx$
(B)	One vertex of a parallelogram is (1, 4), the diagonals intersect at (2, 1) and the sides have slopes 1 and $-\frac{1}{7}$. Find the other three vertices
Q.7-(A)	Solve the differential equation $\sec^2x \tan y dx + \sec^2y \tan x dy = 0$
(B)	Maximize $f(x,y) = x + 3y$ subject to constraints $2x + 5y \leq 30$, $5x + 4y \leq 20$, $x \geq 0$, $y \geq 0$
Q.8-(A)	Find equation of circle passing through A(-7, 7), B(5, -1), C(10, 0)
(B)	Show that mid-point of hypotenuse of a right angle triangle is equidistance from its vertices
Q.9-(A)	If $y = a \cos(\ln x) + b \sin(\ln x)$, Prove that $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$
(B)	Find the centre, foci, eccentricity and vertices of $9x^2 - 12x - y^2 - 2y + 2 = 0$